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On the Conditions which give rise to Surplus in Life Assurance Companies, and on the Amount of the Return, or "Bonus," which such Conditions justify. By CHARLES JELlicoe, V.P.I.A.,
Actuary to the Eagle Insurance Company.

TO trace the succession of events which have given rise to the duplex system of assurance in this country, known as the Participating, and Non-participating, would not be altogether uninteresting or unprofitable. There is little doubt, that the immoderate premiums required at the outset by the few who were disposed to embark in the then novel speculation of life assurance, suggested the idea of mutual association, as a means of achieving the same objects on more advantageous terms; and thus the participating system owed, to a certain extent, its origin to the non-participating.* But a few years' experience soon sufficed to demonstrate to the keen eye of competition, that the payments required under the new arrangements were unnecessarily high; and that, after a large reduction in them, they might be acted upon, not only with safety, but with a considerable profit to boot. Hence arose a second time a series of exclusively proprietary associations, putting forth from time to time rates of premiums "small by degrees and beautifully less," wherewith to bring the many within the attraction of their several spheres. The consequences have been no other than might be expected. Economists have shown, that the existence of poor land is the cause of rent being paid for that which is more productive; and, in like manner, the low rates of some of the Proprietary Companies forced those whose charges were higher to admit the contributors to some share of the advantages which they thus found themselves by contrast in possession of. Nevertheless, the results are somewhat inconsistent: we have societies whose charges are equal, retaining sometimes 50, sometimes 80, per cent. of the surplus; and amongst such as vary in their rates, the proportion returned to the contributors is not unfrequently in the inverse ratio of the average amount of them.

That, under any circumstances, the premiums should be amply sufficient for their purpose, no one will deny—how much more they are than this in a Society for mutual assurance, is not very material, supposing the returns of surplus to be fairly and promptly made; but as regards an exclusively Proprietary Company, it is, of course,

* See Report on application of the "Equitable" for a charter, Vol. I., page 89*.

quite otherwise, and, so far as the assured are concerned, it is obvious that the lower the premiums are, within the limits of safety, the more advantageous are such premiums for them. Between these two types of these institutions, it is probable that the advantages offered may be found to vary, by infinitely small degrees, from great to little, taking into account the rates of premium charged, and the proportion of surplus returned—not to mention the influence which the relative numbers constituting the several Societies now in operation, and their general management, must necessarily bring to bear on the question. But whether the Society be proprietary or mutual, or partly one and partly the other, it is clearly necessary, as I have said, that the premiums charged should be more than sufficient for the risk merely; and the profit in the one case, or the return to the assured in the other, will thus vary with the difference between the total premium charged, and that required for the sum assured, or the risk. Now, I have had occasion in a former paper to point out that the premium for the risk must always depend on the rates of mortality and interest actually prevailing; and further, that the total liability in any Society under its policies at any given time, will be equal to the amount of such premium improved at the actual rate of interest, less the claims; and that the surplus will therefore be equal to the difference between such premium and the premium charged, improved similarly, after deduction of the expenses. In other words, that the liability will be equal to $\int \cdot p'_x - \int \cdot c$, improved as above described; and that the surplus will consist of the amount from year to year of $\int \cdot \phi_x - \int \cdot e$, accumulated in like manner.*

Dismissing, then, the first expression, the results indicated by which we have better means of arriving at, I propose, with the aid of the second, to endeavour to show, in particular cases, and under particular circumstances, what amount of surplus or bonus it is reasonable to expect will arise; and, if possible, to determine the limits beyond which additions to the sum assured become extravagant, or reductions in the premium altogether inconsistent and irrational. For this purpose it will be necessary to assume certain conditions, and to trace the effect of them, and also of such modifications of them, as we may find it expedient with the object in view to adopt. Now the conditions on which the amount of bonus depends are †

* p'_x here denoting the premium for the risk; ϕ_x , the difference between it and the premium charged; c and e , the claims and expenses respectively; and \int being used as a symbol of summation merely.

† There are, no doubt, others of an accidental description—such, for instance, as the abandonment of assurances by their owners; but although this was a matter of frequent

The rates of premium charged ;
 The rates of mortality and interest which actually prevail ;
 The time elapsed from the completion of the assurance ; and,
 The proportion which the yearly expenditure bears to the amount of business annually transacted.

Let us first assume the least favourable of these conditions, and tabulate the results as regards persons commencing their assurances at the ages of 30, 35, 40, 45, and 50 : that is to say,

Let the rates per cent. be as low as £2. 0s. 7d., £2. 6s. 8d., £2. 14s. 10d., £3. 5s. 11d., and £4. 0s. 11d.—let the mortality coincide with that called the experience, and the actual interest be $3\frac{1}{2}$ per cent.—let the expenses be 10 per cent. on the gross amount of premiums annually received ; and, supposing the policies for £1000 each, we shall have the following as the results :—

Age.	Annual Premiums charged.	Annual Premiums required for the Risk.	Difference between the two.	Ten per cent. on Premiums charged.
	£. s. d.	£. s. d.	£. s. d.	£. s. d.
30	20 5 10	17 19 0	13 12 6	14 9 0
35	23 6 8	20 18 8		
40	27 8 4	24 16 4		
45	32 19 2	30 1 0		
50	40 9 2	37 0 8		

Here it will be seen that the surplus is barely sufficient, after addition of interest, to pay the expenses ; and, of course, any bonus at all is out of the question.

Let us now see the effect which a change in the rate of interest only will produce : that is to say, all things else remaining the same, let the rate actually realized be 4 per cent., instead of $3\frac{1}{2}$. Accordingly, we have the following statement :—

Age.	Annual Premiums charged.	Annual Premiums required for the Risk.	Difference between the two.	Ten per cent. on Premiums charged divided equally.	Annual Surplus or Profit.
	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.
30	20 5 10	16 19 5	3 6 5	2 17 9	0 8 8
35	23 6 8	19 17 5	3 9 3	2 17 9	0 11 6
40	27 8 4	23 13 8	3 14 8	2 17 9	0 16 11
45	32 19 2	28 16 10	4 2 4	2 17 9	1 4 7
50	40 9 2	35 15 8	4 13 6	2 17 9	1 15 9
	144 9 2	125 3 0	19 6 2	14 8 9	4 17 5

occurrence formerly, it is no longer so at the present day ; and it is very important to bear in mind, that it is only the value of the policy which in such cases falls into the general fund, not the amount of the premiums paid in respect of it, as the statements frequently put forth would seem to imply.

Here then, it appears, there is an annual surplus or profit of 3·38 per cent. on the premiums charged taken collectively; had the additions to the net premiums been constant, as they ought evidently to be, the profit, or surplus, would have been equal for the year in respect of each assurance. The inequality arises from the additions for expenses, &c. being a per centage on the premium necessary to provide for the risk—an arrangement which is quite inconsistent with either justice or propriety; since, putting apart the item of commission, there is no reason whatever why each of the assured in a given sum should not make the same contribution to the annual surplus and expenses, whether likely to be called upon to do so for a short period, or for many years to come. In the above statement, the expenses are at the rate of 10 per cent.; had they been half that rate, it will be observed that the surplus, after payment of them, would be increased to £12. 1s. 10*d.*, or nearly 8·5, instead of 3·38, per cent. on the premiums.

I will now assume conditions of a more favourable character, and such as are not always attained simultaneously in practice, although it may happen that one or another in particular may occasionally be of a more advantageous kind than that now proposed. Accordingly, the following table exhibits the results to be looked for in a Society charging the Northampton Premiums, as they are called; but in which the rate of mortality actually prevailing coincides with that deduced by Mr. Griffith Davies from the experience of the “Equitable;” and the real rate of interest is 4 per cent.; and in which the expenses do not exceed 5 per cent. on the amount of annual premiums received:—

Age.	Annual Premium charged.	Annual Premium required for the Risk.	Difference between the two.	Five per cent. on Premiums charged, divided equally.	Annual Surplus or Profit.	Quinquennial Surplus or Profit.
	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	
30	26 13 5	18 0 10	8 12 7	1 14 11	6 17 8	38·771
35	29 18 3	20 16 5	9 1 10	1 14 11	7 6 11	41·380
40	33 19 6	24 5 6	9 14 0	1 14 11	7 19 1	44·815
45	38 18 9	28 17 10	10 0 11	1 14 11	8 6 0	46·754
50	45 6 0	35 1 5	10 4 7	1 14 11	8 9 8	47·784
	174 15 11	127 2 0	47 13 11	8 14 7	38 19 4	

From this we learn, that the annual surplus or profit arising under the circumstances described, would be at the rate of 22·5 per cent., very nearly, on the gross premiums; and this, it will be remarked, is, as in the former instances, exclusive of interest accruing in the interval between the time of the several payments,

and the end of the financial year. It has been scarcely worth while to introduce that item hitherto; but for the matters I am now about to discuss, viz., the additions to the sum assured which this annual surplus will suffice for, or the reductions in the premium which it will warrant, it must, of course, be taken into consideration.

Now the interval more frequently chosen than any other for the surplus to accumulate, and at the expiration of which the division of it is to be made, is five years, when either a fifth or proportionate part of the whole is applied in reduction of the premium next falling due (a similar process being gone through annually thereafter, and an annual valuation being made for the purpose); or the whole accumulation is distributed at once over all the future payments, or a given number of them; or it is converted into a reversionary sum, payable when the amount assured itself becomes payable, leaving nothing more, of course, to be divided, till another term of five years has elapsed. Now the last column in the foregoing statement shows the amount, at the end of five years, of the annual surplus, improved at the prevailing rate of interest; and on the supposition that all the payments are made at the beginning of the year, and assuming the conditions above laid down to hold good, it will be found that the amounts in question will justify the distribution of the following alternative benefits: viz.—

Age.	Proportion of Surplus applied annually, after first 5 years, in reduction of Premium.	Whole quinquennial Surplus applied in reduction of all the future payments.			Whole Quinquennial Surplus applied in augmentation of the Amount assured.		
		At end of 5 years.	At end of 10 years.	At end of 15 years.	At end of 5 years.	At end of 10 years.	At end of 15 years.
30	8.274	2.357	4.789	7.402	100.	192.	275.
35	8.963	2.595	5.383	8.425	98.	186.	267.
40	9.704	3.020	6.315	9.943	95.	182.	262.
45	10.126	3.438	7.223	11.454	92.	176.	254.
50	10.350	3.867	8.191	13.147	86.	166.	239.

The sums in the first money column of the above table are nothing more than the annual differences in each case, improved at the actual rate of interest for five years; and represent the reductions to be made under the system which allows a return of a certain portion of the premium annually after the first five years have elapsed. This is probably the most simple and correct view to take of this method; since it regards the assured as receiving, at the end of the fifth and every subsequent year, the surplus or dif-

ference, with interest for the interval, of the premium paid five years before—four years' differences and interest upon them being thus always in arrear. The other benefits are arrived at in the usual way; but an addition is made to the age, where the sum assured is increased, corresponding with about one-sixth of the expectation in each case, to counteract any influence which the power of selection afforded in this plan may possibly be found to exercise.

Here then, it will be seen that the reduction after the first five years, for the remainder of life, in the annual premium, varies with the age from 30 to 23 per cent. of the premium charged—the reduction by successive quinquennial gradations of course, although not ostensibly, nearly corresponding. The sums added to the amount assured at the first division range from 10· to 8·6 per cent. upon it—after the second, from 19·2 to 16·6 per cent.—and after the third, from 27·5 to 23·9 per cent. I have supposed three quinquennial divisions to afford more extended means of comparison, and have assumed the accumulation at each to be of the same amount—as it obviously would be, the conditions being unaltered.

Such, then, are the advantages which may reasonably be looked for under the circumstances I have described; and hence, should we find in any given instance that they are exceeded, we must be unavoidably led to the conclusion, either that some error or that an extraneous source of profit exists, or that some or all of the conditions I have alluded to are of a still more favourable character than those here assumed: to wit, that more than 4 per cent. is realized, that the mortality is less than that experienced amongst the members of the "Equitable" Society, or that the expenses do not amount to 5 per cent. per annum on the premiums received.

But it may be, that the premiums required are at a still higher rate than those generally known as the Northampton. Instances of this are very rare; nevertheless, it will be interesting to inquire how far such increased rate will affect the results under consideration; and I will therefore give one more statement, based upon the same data as I have adopted in the foregoing one, except that the premiums, instead of being those called the Northampton, are at rates as high as £2. 19s., £3. 6s., £3. 15s., £4. 6s. 6d., and £5. 4s., per cent.: videlicet,—

Age.	Annual Premium charged.	Annual Premium required for the Risk.	Difference between the two.	Five per cent. on Premiums charged, divided equally.	Annual Surplus or Profit.	Amount of the foregoing at end of 7 years.
	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.
30	29 10 0	18 0 10	11 9 2	1 19 0	9 10 2	12 11 0
35	33 0 0	20 16 5	12 3 7	1 19 0	10 4 7	13 10 0
40	37 10 0	24 5 6	13 4 6	1 19 0	11 5 6	14 17 8
45	43 5 0	28 17 10	14 7 2	1 19 0	12 8 2	16 7 6
50	52 0 0	35 1 5	16 18 7	1 19 0	14 19 7	19 15 6
	195 5 0	127 2 0	68 3 0	9 15 0	58 8 0	77 1 8

The annual profit here shown, exclusive of interest, is £58. 8s. in respect of £195. 5s. premiums, or very nearly 30 per cent. ; and if we suppose it in each case to remain in abeyance, and at interest for seven years, it will be seen by the results in the last column that, at the end of that time, the accumulated difference will suffice for a reduction in the original premium, varying with the age, from 43 to 38 per cent. of it ; and since in each succeeding year the same sum will accrue from the difference of the premium paid seven years previously, a similar reduction may evidently be made annually throughout the remaining term of life, provided, of course, that the prescribed conditions continue to obtain. This rate of reduction appears a high one : on the other hand, it is to be remembered that, in addition to the extravagant premium at first demanded, a long period elapses before any return at all is made ; and then, although the per centage of the reduction is large, the sum remaining to be paid is still, in consequence of the largeness of the original rate, considerable. It is, however, worthy of remark, that in all these modes of division room is left for some increase in after years in the rate of the return itself. It will be observed, for instance, in this one, that at the decease of each of the lives assured for seven years and upwards, an arrear of no less than six years' differences, with interest upon them, will fall into the general fund ; whilst, for such of the lives as drop within the term of seven years, the Society will receive the full annual premium, without any deduction whatever. Under the system which allots the share of surplus by way of addition to the sum assured, the holder of the policy will get the whole amount of his share, if the life drop immediately after the allotment of it : otherwise some arrears will, as in the case before mentioned, fall into the general fund at the decease of the life assured, unless a proportionate return is made for the interval, as at the present day is commonly done.

The instances I have now cited will, I trust, serve to give at least a general idea of the sources from which surplus is derived, and of the limits beyond which any stated amount of it must be considered as extravagant. The circumstances of associations for life assurance are no doubt of an infinitely varied character, and some are aided by coincidences which others are not so fortunate as to experience ; nevertheless, all must more or less be controlled by the considerations we have been discussing, and in the long run will, there is every reason to believe, exhibit a marked conformity to them—unless indeed, persevering in a wilful neglect of the natural laws by which they are governed, they bring on a premature decay, and fall into disorder and confusion.

Before concluding this paper, I would once more advert to the inconveniences which arise from the manner in which the premiums for life assurance are “loaded,” as it is termed, and especially to the extraordinary anomalies to be found in setting forth the participating and non-participating scales. Sometimes the differences between the two are not far from constant ; at others, they increase as the age increases ; and instances are even not unfrequent where the reverse takes place, and the differences between the two actually decrease from the commencement of the table to its close. In a purely mutual association, the manner of “loading” is not very material, since any inaccuracies in it can be rectified in allotting the return of the surplus ; and although it is clearly desirable that the members should be called upon to contribute on this score, as nearly as possible, in an equal proportion, a perfect equality could not probably be established at setting out, since it would not be possible to predict with exactness the rate of premium for the risk, nor would this last always continue the same, even were such prediction at first possible. But although this is of minor importance in the mutual system, it is by no means so in the proprietary, where no return is made, and where of course the inequalities I am speaking of are irremediable. Here it is of most material consequence that the premiums should be adjusted fairly, and that as little deviation as possible from one common measure should be allowed in the rate of extra contribution ; unfortunately, however, this point appears to be very little regarded, and hence a good deal of injustice is, no doubt, unwittingly done. Nor is this the only instance in which an erroneous practice continues to prevail : notwithstanding what has been said and written on the subject there is reason to believe that the old and inaccurate methods of determining surplus, and of dividing and allotting it, are all for the most part very

generally persevered in,* creating a confusion and entanglement which no skill or penetration would now suffice to unravel, and perpetrating an amount of injustice which must be enormous, unless it happens, by a fortunate concurrence of circumstances, that in the multitude of errors thus occasioned, the positive and negative ones nearly balance each other. As this perseverance is, no doubt, to be attributed to a belief in the accuracy of the methods I am condemning, I propose, having now laid down what I believe to be the true ones, to investigate on future occasions the nature and character of those methods, and to endeavour to demonstrate the mischievous and injurious consequences resulting from them.

* Take, for instance, the generally prevailing method of dividing the surplus in the proportion indicated by the difference between the amount of premiums paid at compound interest and the value of the assurance. That is to say, in the proportion shewn by $f.p_x - f.l$.

Since $f.l = f.p'_x - f.c$, the former expression is equal to $f.p_x - (f.p'_x - f.c) = f.p_x + f.c$; so that the surplus is thus actually allotted amongst the assurances by this method, not simply in the proportion in which each has contributed thereto, but in the extraordinary ratio of its contribution to surplus, and to the payment of claims.

On the Uniform Action of the Human Will, as exhibited by its Mean Results in Social Statistics. By SAMUEL BROWN, Esq.,
one of the Honorary Secretaries of the Institute of Actuaries.

[Read before the Institute of Actuaries, 31st May, 1852.]

IT has frequently been remarked, that however varied and uncertain may be the occurrence of the events to which the life of a single individual is exposed, the average return in a large mass of persons is so regular, as to be predicted with confidence within very small limits of error. On the uniform happening of these events under similar circumstances, or the discovery of the laws by which they are governed, the actuary depends for the application of the theories of probabilities to the many useful purposes for which they may be rendered available. The principal difficulty which he has to encounter is the indefinite character of the event itself. In endeavouring to ascertain the law of mortality at any period of time, or in any particular country, the event is certain and definite whenever it does occur. The attention is only required to the difference of the circumstances which cause it—the age of the party, the nature of the disease, or the locality within certain